Teacher notes

Topic D

Work done in a field – more on $W = m\Delta V_g$ and $W = q\Delta V_e$.

We teach that the expression $W = m\Delta V_g$ can be used to calculate the work done when a point mass *m* is moved from one point in a gravitational field to another. But this is very confusing to students especially when they face a question such as:

A satellite of mass *m* orbits a planet in a circular orbit of radius *R*. What work must be done to make the satellite orbit the planet in an orbit of radius 2*R*?

The naïve approach is to use $W = m\Delta V_g$ and therefore say $W = m\left(-\frac{GM}{2R} - \left(\frac{GM}{R}\right)\right) = +\frac{GMm}{2R}$ which is the wrong answer. Why?

The formula $W = m\Delta V_g = m\left(V_2 - V_1\right)$ is used to calculate the work done by an external agent in moving a point mass *m* from a point 1 where the gravitational potential is V_1 to another point 2 where the potential is V_2 *under the condition that the point mass is moved at a constant and infinitesimally small speed*. Thus, the formula $W = m\Delta V_g$ cannot be used to answer the problem above since the speed changes.

A correct approach would be to use the fact that the total energy of the satellite in the two orbits is different. The difference in total energy is the work that must be done on the satellite by an external agent (the engines of the satellite for example). Since $E_T = -\frac{GMm}{2r}$, this would give $W = \left(-\frac{GMm}{4R} - \left(\frac{GM}{2R}\right)\right) = +\frac{GMm}{4R}$ which is the correct answer.

See the answer to part (c) of Teacher Notes D8 for a specific calculation of the engine work which leads to the change in total energy.

Exam questions are notoriously loose when they ask about work done. One must always ask what is the work done by a particular force. So consider the standard question that asks for the work done when a charge of +2.0 μC is moved from a point X where the potential is 20 V to a point Y where the potential is 40 V. Are we asking for the work done by an external agent who will move the charge from X to Y or are we asking for the work done by the electric force as the charge is moved from X to Y? Because the two are not the same. The question must also specify that the charge is moved from X to Y at a small constant speed. Let W_F be the work done by the electric force and W_E the work done by the external agent. By the work-kinetic energy relation, $W_{net} = \Delta E_K = 0$ and $W_{net} = W_F + W_E$. Hence, $W_F = -W_E$. We know that $W_E = q\Delta V = 2.0\times 10^{-6}\times (40-20) = 40\ \mu J$ and so $W_F = -40\ \mu J$. In other words,

Work done by external agent $W = q\Delta V = q(V_{final} - V_{initial})$

Work done by field $W = -q\Delta V = -q(V_{final} - V_{initial})$

Now consider a different problem: the charge (2.0 μC) is accelerated from rest at Y to a speed *v* by the time it gets to X. The mass of the charge is 3.2 mg. What is *v*?

We use the work-kinetic energy relation.

$W_{net} = \Delta E_K$

The net force is the electric force and so (from what we learned above)

$W_{net} = -q\Delta V = -q(V_{final} - V_{initial}) = -2.0\times (20-40) = +40\ \mu J$

Hence

$40\times 10^{-6} = \frac{1}{2}\times 3.2\times 10^{-6}\times v^2$ giving $v = 5.0\ m\ s^{-1}$.